Networks and Link Analysis

Web Anchor Text
The Web as a Directed Graph

Assumption 1:
A hyperlink between pages denotes author perceived relevance (quality signal)

Assumption 2:
The anchor of the hyperlink describes the target page (textual context)
Anchor Text

- For *ibm* how to distinguish between:
  - IBM’s home page (mostly graphical)
  - IBM’s copyright page (high term frequency for “ibm”)
  - Rival’s spam page (arbitrarily high term frequency)

A million pieces of anchor text with “ibm” send a strong signal
Indexing anchor text

When indexing a document $D$, include anchor text from links pointing to $D$.

Armonk, NY-based computer giant IBM announced today

www.ibm.com
Solutions, Services, Products, MyIBM

Question Answering Systems: Apple’s Siri IBM’s Watson

Big Blue today announced record profits for the quarter
Indexing anchor text

• Can sometimes have unexpected side effects –
  • Google bombing

• Can score anchor text with weight depending on the authority of the anchor page’s website
  • E.g., if we were to assume that content from cnn.com or yahoo.com is authoritative, then trust the anchor text from them
Many NLP Applications of Anchor Text

• Finding synonyms

• Finding translations of named entities

• Providing constituent boundaries for parsers
Networks and Link Analysis

Web Anchor Text
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PageRank: Overview and Markov Chains
Combining link structure with text

• A good search result looks at more than just query-document text overlap

• One factor: page *popularity*.
  • Pages that are pointed to by lots of other pages are popular.
  • We can use link counts as a measure of static goodness,
  • Combine link counts with the text match score
Using link structure to measure page importance

• Simplest: use link counts as popularity measure
  • Undirected popularity:
    • Page score = degree: the number of in-links plus the number of out-links (3+2=5).
  • Directed popularity:
    • Page score = number of in-links (3).
Spamming simple popularity

- Simple popularity heuristics can be spammed to give your page a high score, whether it’s:
  - the number of in-links plus the number of out-links
  - number of in-links
C has higher PageRank than E, even though E has more inlinks
PageRank scoring

- Imagine a browser doing a random walk on web pages:
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- “In the steady state” each page has a long-term visit rate - use this as the page’s score.
Not quite enough

• The web is full of dead-ends.
  • Random walk can get stuck in dead-ends.
  • Makes no sense to talk about long-term visit rates.
Teleporting

• At a dead end, jump to a random web page.
• At any non-dead end, with probability 10%, jump to a random web page.
  • With remaining probability (90%), go out on a random link.
• 10% - a parameter.
Result of teleporting

- Now cannot get stuck locally.
- There is a long-term rate, the Pagerank, at which any page is visited
Markov chains

• A Markov chain:
  • \( N \) states,
  • An \( N \times N \) transition probability matrix \( P \).
• At each step, we are in exactly one of the states.
• For \( 1 \leq i,j \leq n \), the matrix entry \( P_{ij} \) tells us the probability of \( j \) being the next state, given we are currently in state \( i \).

\[
\sum_{j=1}^{n} P_{ij} = 1.
\]
Markov chains

- Transition probability matrix $P$

$$P = \begin{pmatrix}
0 & 0.5 & 0.5 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}$$
Random Surfers and Markov chains

• Markov chains are abstractions of random walks.
• Each state
  • represents one web page
• Each transition probability
  • represents the probability of moving from one page to another
• We can derive the transition probability $P$ from the adjacency matrix $A$ of the web graph.
Teleporting, more formally

• If a node has no out-links, the random surfer teleports:
  • the transition probability to each node in the N-node graph is 1/N
• If a node has K>0 outgoing links:
  • with probability $0 < \alpha < 1$ the surfer teleports to a random node
    • probability is $\alpha/N$
  • with probability $1-\alpha$ the surfer takes a normal random walk
    • probability is $(1-\alpha)/K$
Deriving transition probability matrix $P$ from adjacency matrix $A$

- $A$ is the adjacency matrix of the web graph
  - $A_{ij}$ is 1 if there is a hyperlink from page $i$ to page $j$

- If a row of $A$ has no 1's, then replace each element by $1/N$.
  For all other rows proceed as follows.
- Divide each 1 in $A$ by the number of 1's in its row. Thus, if there is a row with three 1's, then each of them is replaced by $1/3$
- Multiply the resulting matrix by $(1-\alpha)$
- Add $\alpha/N$ to every entry of the resulting matrix, to obtain $P$. 
Computing P with teleportation

\[
P_{\alpha=0} = \begin{pmatrix}
0 & 1 & 0 \\
0.5 & 0 & 0.5 \\
0 & 1 & 0
\end{pmatrix}
\]

\[
P_{\alpha=0.5} = \begin{pmatrix}
1/6 & 2/3 & 1/6 \\
5/12 & 1/6 & 5/12 \\
1/6 & 2/3 & 1/6
\end{pmatrix}
\]

\[
P[1,*] = (1-\alpha) (0 1 0) + \alpha (1/N 1/N 1/N)
\]

\[
P[1,*] = 0.5 (0 1 0) + 0.5(1/3 1/3 1/3)
\]
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PageRank: Computation
Computing PageRank:
The probability of being in a state

- A probability (row) vector \( \mathbf{x} = (x_1, \ldots, x_n) \) tells us where the walk is at any point.
- E.g., \( (000\ldots1\ldots000) \) means we’re in state \( i \).

More generally, the vector \( \mathbf{x} = (x_1, \ldots, x_n) \) means the walk is in state \( i \) with probability \( x_i \).

\[
\sum_{i=1}^{n} x_i = 1
\]
Computing PageRank:
Change in probability vector

• If the probability vector is \( \mathbf{x} = (x_1, \ldots, x_n) \) at this step, what is it at the next step?
• Recall that row \( i \) of transition matrix \( \mathbf{P} \) tells us where we go next from state \( i \).
• So from \( \mathbf{x} \), our next state is distributed as \( \mathbf{x} \mathbf{P} \).
Ergodic Markov chains

- A Markov chain is **ergodic** if
  - you have a path from any state to any other
  - For any start state, after a finite transient time $T_0$, the probability of being in any state at a fixed time $T>T_0$ is nonzero.

Not ergodic (even/odd).
Ergodic Markov chains

• For any ergodic Markov chain, there is a unique long-term visit rate for each state.
  • A steady-state probability distribution \( \pi = (\pi_1, \ldots, \pi_n) \).
  • Over a long time-period, we visit each state in proportion to this rate.
    • Thus \( \pi_i \) is the PageRank of state \( i \).
• It doesn’t matter where we start.
Steady state example

- The steady state looks like a vector of probabilities $\pi = (\pi_1, \ldots, \pi_n)$:
  - $\pi_i$ is the probability that we are in state $i$.

For this example, $\pi_1 = \frac{1}{4}$ and $\pi_2 = \frac{3}{4}$.
How do we compute this vector?

- Let $\pi = (\pi_1, \ldots, \pi_n)$ denote the row vector of steady-state probabilities.
- If our current position is described by $\pi$, then the next step is distributed as $\pi P$.
- But $\pi$ is the steady state, so $\pi = \pi P$.
- Solving this matrix equation gives us $\pi$.
  - So $\pi$ is the (left) eigenvector for $P$.
  - (Corresponds to the “principal” eigenvector of $P$ with the largest eigenvalue.)
  - Transition probability matrices always have largest eigenvalue 1.
The power iteration method of computing $\pi$

- Recall, regardless of where we start, we eventually reach the steady state $\pi$.
- Start with any distribution (say $x=(10...0)$).
- After one step, we’re at $xP$;
- after two steps at $xP^2$, then $xP^3$ and so on.
- “Eventually” means for “large” $k$, $xP^k = \pi$.
- Algorithm: multiply $x$ by increasing powers of $P$ until the product looks stable.
Example of power iteration

Let's say surfer starts in state 1:

\[ \bar{x}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

\[ \bar{x}_1 = \bar{x}_0 P = \begin{pmatrix} 1/6 & 2/3 & 1/6 \end{pmatrix} \begin{pmatrix} 1/6 & 2/3 & 1/6 \\ 5/12 & 1/6 & 5/12 \\ 1/6 & 2/3 & 1/6 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} \]
Power iteration example (continued)

<table>
<thead>
<tr>
<th>$\vec{x}$</th>
<th>Node 1 PageRank</th>
<th>Node 2 PageRank</th>
<th>Node 3 PageRank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{x}_0$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\vec{x}_1$</td>
<td>1/6</td>
<td>2/3</td>
<td>1/6</td>
</tr>
<tr>
<td>$\vec{x}_2$</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>$\vec{x}_3$</td>
<td>1/4</td>
<td>1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>$\vec{x}_4$</td>
<td>7/24</td>
<td>5/12</td>
<td>7/24</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\vec{x} = \vec{\pi}$</td>
<td>5/18</td>
<td>4/9</td>
<td>5/18</td>
</tr>
</tbody>
</table>

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PageRank summary

- **Preprocessing:**
  - Given graph of links, build matrix \( P \).
  - From it compute the PageRank vector \( \pi \).
  - The PageRank of page \( i \), \( \pi_i \), is between 0 and 1

- **Query processing:**
  - Retrieve pages meeting query.
  - Rank them by their PageRank.
  - Order is query-independent.
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PageRank: Computation